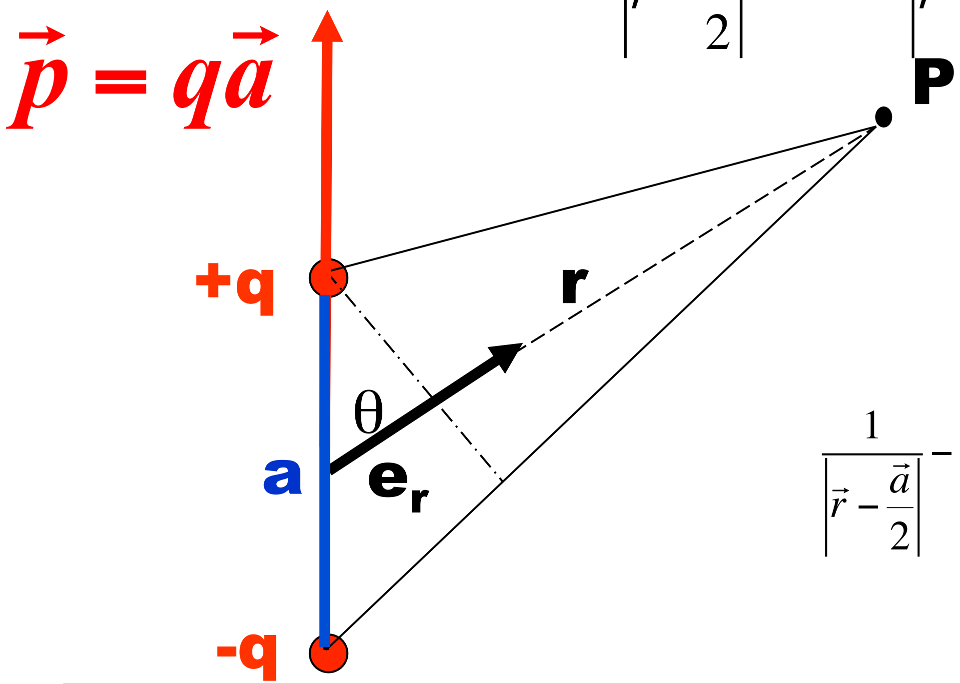
Il dipolo elettrico

- Dipolo elettrico
- Interazioni del dipolo elettrico
- Sviluppo in serie di multipoli
- Esempi

Dipolo elettrico

- Due cariche puntiformi **+q** e **–q** distanti ***a***

$$V_P = \frac{q}{4\pi\varepsilon_0}\frac{1}{\left|\vec{r}-\frac{\vec{a}}{2}\right|} - \frac{q}{4\pi\varepsilon_0}\frac{1}{\left|\vec{r}+\frac{\vec{a}}{2}\right|} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left|\vec{r}-\frac{\vec{a}}{2}\right|} - \frac{1}{\left|\vec{r}+\frac{\vec{a}}{2}\right|}\right]$$



P "lontano" a/r << 1

$$f(\vec{r}+\vec{h}) \approx f(\vec{r}) + \vec{h}\cdot\vec{\nabla}f + \cdots$$

$$\frac{1}{\left|\vec{r}-\frac{\vec{a}}{2}\right|} - \frac{1}{\left|\vec{r}+\frac{\vec{a}}{2}\right|} \approx \left(\frac{1}{r} - \frac{\vec{a}}{2}\cdot\vec{\nabla}\frac{1}{r}\right) - \left(\frac{1}{r} + \frac{\vec{a}}{2}\cdot\vec{\nabla}\frac{1}{r}\right) = -\vec{a}\cdot\vec{\nabla}\frac{1}{r} = \frac{\vec{a}\cdot\vec{e}_r}{r^2}$$

Dipolo elettrico

$$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left|\vec{r}-\frac{\vec{a}}{2}\right|}-\frac{1}{\left|\vec{r}+\frac{\vec{a}}{2}\right|}\right] \approx \frac{q}{4\pi\varepsilon_0}\frac{\vec{a}\cdot\vec{e}_r}{r^2} = \frac{\vec{p}\cdot\vec{e}_r}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V(r) = \frac{\vec{p}\cdot\vec{e}_r}{4\pi\varepsilon_0 r^2}}$$

$$\vec{E} = -\vec{\nabla}\left(\frac{\vec{p}\cdot\vec{e}_r}{4\pi\varepsilon_0 r^2}\right)$$

Va a zero piu` rapidamente del potenziale di una carica puntiforme (+q, -q tendono a neutralizzarsi)

$$\propto \frac{1}{r^3}$$

Campo elettrico del dipolo

$$\vec{e}_r = \frac{\vec{r}}{r}$$

$$\vec{E} = -\vec{\nabla}\left(\frac{\vec{p}\cdot \vec{e}_r}{4\pi\varepsilon_0 r^2}\right) = -\frac{1}{4\pi\varepsilon_0}\vec{\nabla}\left(\frac{\vec{p}\cdot\vec{r}}{r^3}\right)$$

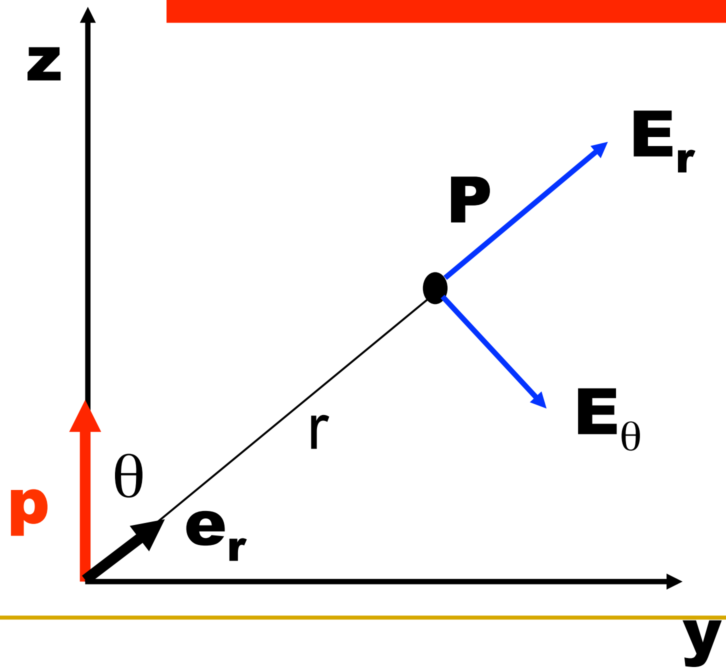
$$= -\frac{1}{4\pi\varepsilon_0}\frac{\vec{\nabla}(\vec{p}\cdot\vec{r})r^3 - 3r^2(\vec{p}\cdot\vec{r})\vec{\nabla}r}{r^6}$$

$$\vec{\nabla}\left(\vec{p}\cdot\vec{r}\right) = \vec{p}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{r^3\vec{p} - 3r^2(\vec{p}\cdot\vec{r})\vec{e}_r}{r^6} = -\frac{1}{4\pi\varepsilon_0 r^3}\left[\vec{p} - 3(\vec{p}\cdot\vec{e}_r)\vec{e}_r\right]$$

Campo elettrico del dipolo

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p}\right]}$$



- Giace nel piano individuato da $\vec{p}$ ed $\vec{e}_r$
- Decresce con la distanza come r^{-3}
- Nel piano mediano $(\vec{p}\cdot\vec{e}_r = 0)$ $\vec{E} \propto -\vec{p}$
- Lungo l'asse del dipolo $\vec{E} \propto \vec{p}$
- Negli altri casi mai parallelo a $\vec{p}$

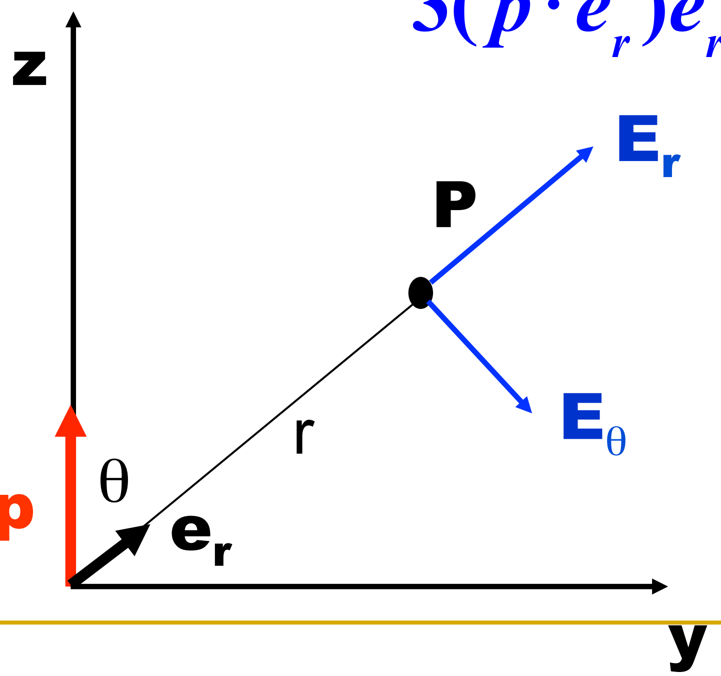
Campo elettrico del dipolo

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p}\right]$$

$$\vec{p} = p\cos\theta\,\vec{e}_r - p\sin\theta\,\vec{e}_\vartheta$$

$$\vec{p}\cdot\vec{e}_r = p\cos\theta$$

$$3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p} = 2p\cos\theta\,\vec{e}_r + p\sin\theta\,\vec{e}_\theta$$



Campo elettrico del dipolo

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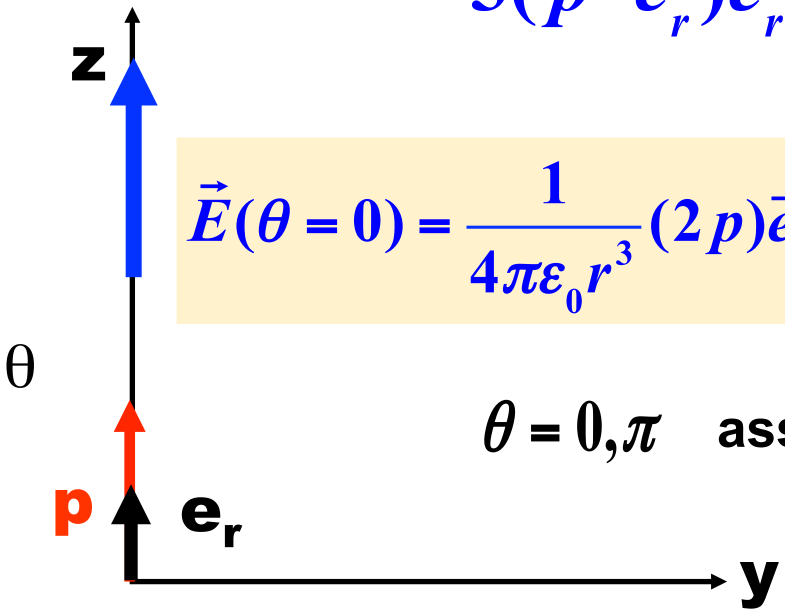


$$\boxed{\vec{E}(\theta = 0) = \frac{1}{4\pi\varepsilon_0 r^3}(2p)\vec{e}_r}$$

$\theta = 0, \pi$ asse del dipolo: campo radiale, // dipolo

Campo elettrico del dipolo

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p}\right]$$

$$\vec{p} = p\cos\theta\vec{e}_r - p\sin\theta\vec{e}_\vartheta$$

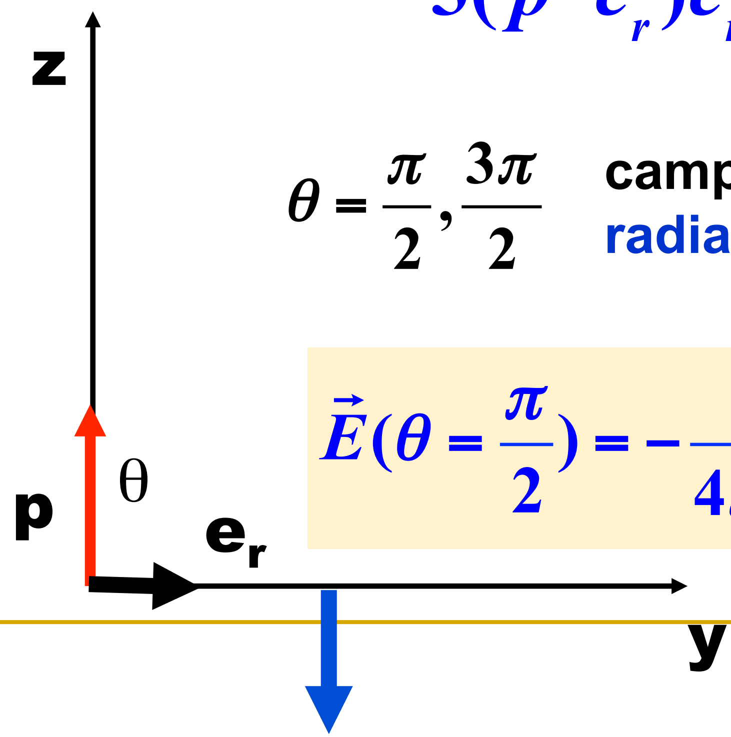
$$\vec{p}\cdot\vec{e}_r = p\cos\theta$$

$$3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p} = 2p\cos\theta\vec{e}_r + p\sin\theta\vec{e}_\theta$$

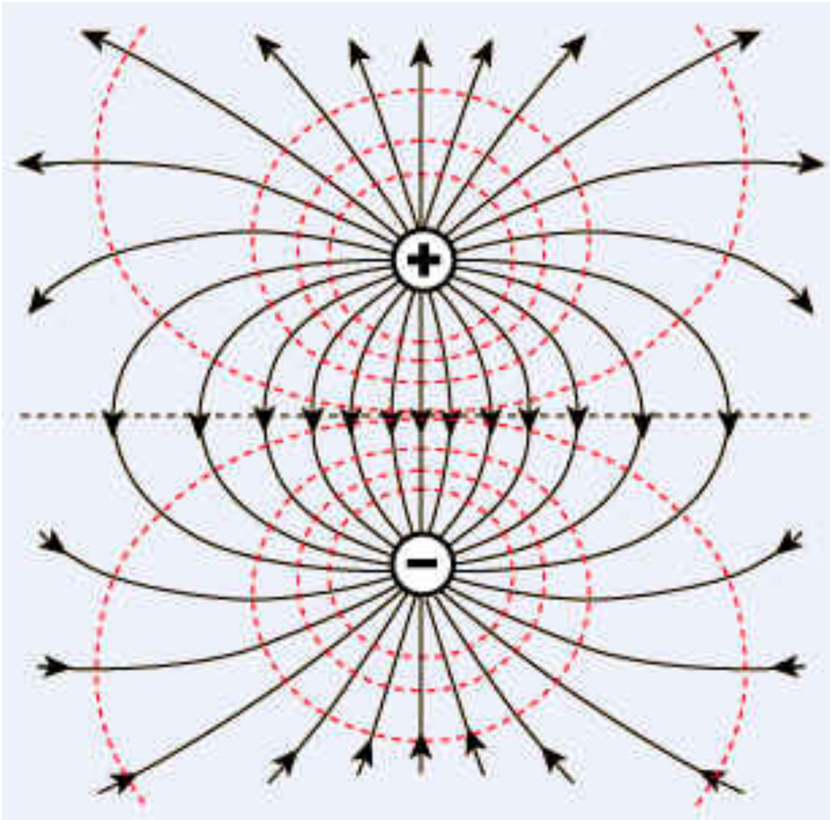
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

campo perpendicolare alla direzione radiale, // dipolo

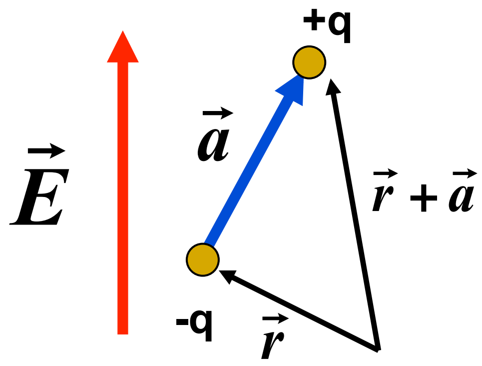
$$\vec{E}(\theta = \frac{\pi}{2}) = -\frac{1}{4\pi\varepsilon_0 r^3}p\vec{e}_z$$



Linee di forza ed equipotenziali



Dipolo elettrico in un campo elettrostatico

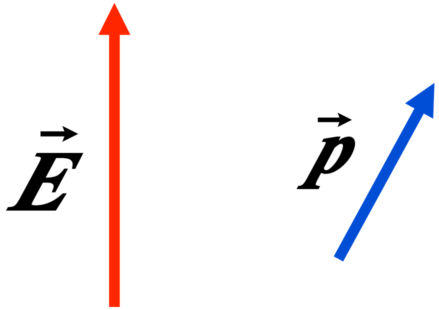


Energia potenziale del dipolo

$$U(\vec{r}) = qV(\vec{r}+\vec{a}) + [-qV(\vec{r})] = q\left[V(\vec{r}+\vec{a}) - V(\vec{r})\right]$$

$$\simeq q\underbrace{\vec{a}}_{\vec{p}} \cdot \underbrace{\vec{\nabla}V(\vec{r})}_{-\vec{E}} \qquad \Rightarrow \boxed{U(\vec{r}) = -\vec{p}\cdot\vec{E}}$$

Dipolo elettrico in un campo elettrostatico



Forza

$$q\vec{a} = \vec{p}$$

$$\vec{F} = \vec{F}(\vec{r}) + \vec{F}(\vec{r}+\vec{a}) = -q\vec{E}(\vec{r}) + q\vec{E}(\vec{r}+\vec{a}) = q\left[\vec{E}(\vec{r}+\vec{a}) - \vec{E}(\vec{r})\right] \simeq q\left(\vec{a}\cdot\vec{\nabla}\right)\vec{E}$$

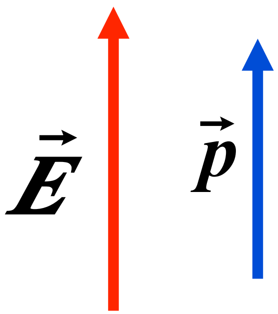
$$\boxed{\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}}$$

$$F_i = \left(\vec{p}\cdot\vec{\nabla}\right)E_i = \left(p_k \partial_k\right)E_i$$

Anche da:

$$\vec{F} = -\vec{\nabla}U = -\vec{\nabla}(-\vec{p}\cdot\vec{E}) = \vec{\nabla}(\vec{p}\cdot\vec{E})$$

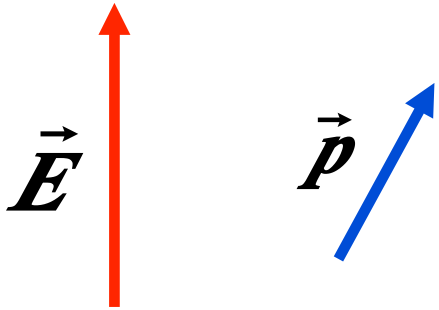
$$\vec{\nabla}\times\vec{E} = 0$$



$$\vec{p}\cdot\vec{E} = pE$$

$$\vec{F} = p\vec{\nabla}E$$

Dipolo elettrico in un campo elettrostatico



Forza

$$\boxed{q\vec{a} = \vec{p}}$$

$$\vec{F} = \vec{F}(\vec{r}) + \vec{F}(\vec{r}+\vec{a}) = -q\vec{E}(\vec{r}) + q\vec{E}(\vec{r}+\vec{a}) = q\left[\vec{E}(\vec{r}+\vec{a}) - \vec{E}(\vec{r})\right] \simeq q\left(\vec{a}\cdot\vec{\nabla}\right)\vec{E}$$

$$\boxed{\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}}$$

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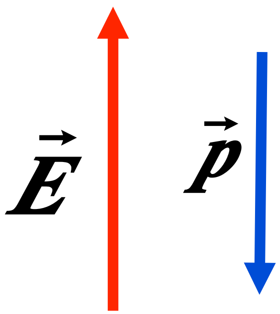
Anche da:

$$\vec{F} = -\vec{\nabla}U = -\vec{\nabla}\left(-\vec{p}\cdot\vec{E}\right) = \vec{\nabla}\left(\vec{p}\cdot\vec{E}\right)$$

$$\vec{\nabla}\times\vec{E} = 0$$



$$\vec{p}\cdot\vec{E} = -pE$$

$$\vec{F} = -p\vec{\nabla}E$$

Dipolo elettrico in un campo elettrostatico uniforme

- La forza e` nulla: $\vec{F} = p\underbrace{\vec{\nabla}E}_{=0}$

- Momento della forza:

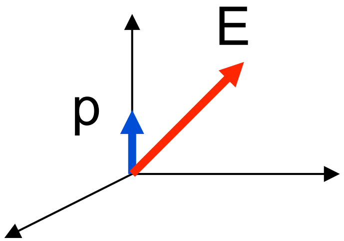
$$\vec{M} = \vec{r} \times \vec{F}_{-q} + (\vec{r} + \vec{a}) \times \vec{F}_{+q} \qquad \vec{F}_{\pm q} = \pm q\vec{E}$$

$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$

Il momento fa ruotare il dipolo per allinearlo al campo elettrostatico.

Minimo dell' energia di interazione.

Dipolo elettrico in un campo elettrostatico uniforme



$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$

$$\vec{p} \times \vec{E} = p\vec{e}_z \times (E_y\vec{e}_y + E_z\vec{e}_z) = -pE\sin\theta\ \vec{e}_x$$

$$M = pE\sin\theta = -\frac{dU}{d\theta} \qquad dW = Md\theta = -dU$$

$(U = -pE\cos\theta)$

Interazione tra due dipoli

Consideriamo il primo dipolo nel campo del secondo, o viceversa: il problema e` simmetrico.

Energia di interazione $\propto r^{-3} \quad \Rightarrow \quad F \propto r^{-4}$

$$U_{12} = -\vec{p}_1 \cdot \vec{E}_2 = -\frac{1}{4\pi\varepsilon_0 r^3}\vec{p}_1 \cdot \left(3(\vec{p}_2 \cdot \vec{e}_r)\vec{e}_r - \vec{p}_2\right)$$

$$\boxed{U_{12} = \frac{1}{4\pi\varepsilon_0 r^3}\left(\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \vec{e}_r)(\vec{p}_2 \cdot \vec{e}_r)\right) = U_{21}}$$