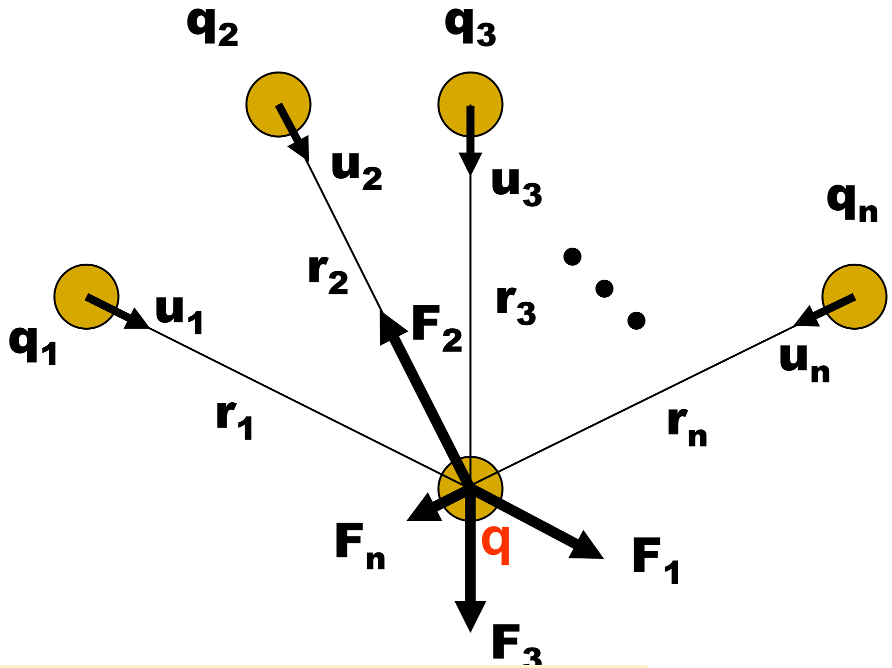
Principio di sovrapposizione

Forza su **q**



$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q q_i}{r_i^2}\vec{u}_i$$

$$\vec{F} = \sum_i \vec{F}_i$$

Linearita` nella carica elettrica

I versori $\mathbf{u}_i$ vanno dalla sorgente alla carica su cui si valuta la forza

Campo elettrostatico

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

carica positiva di prova

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Campo elettrostatico

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

carica positiva di prova

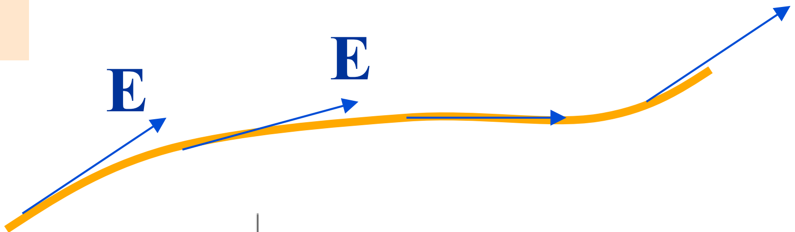
$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i^2} \vec{u}_i$$

$$\vec{F}(\vec{x}) = q\,\vec{E}(\vec{x})$$

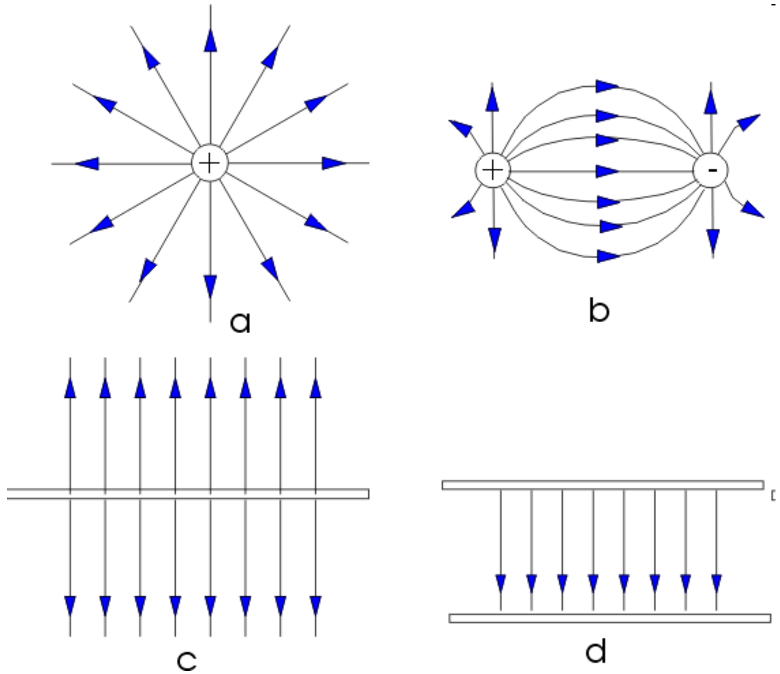
Campo elettrostatico

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

carica positiva di prova



$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i^2} \vec{u}_i$$



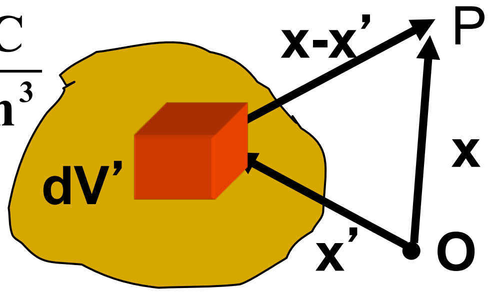
$$\vec{F}(\vec{x}) = q \, \vec{E}(\vec{x})$$

Distribuzioni continue di carica

- Campo macroscopico → media dei campi microscopici a distanze >> atomiche (~10^{-10} m)
- Carica "infinitesima" dal punto di vista macroscopico – carica variabile **continua**

$$Q = \int \rho(\vec{x'}) d^3x'$$

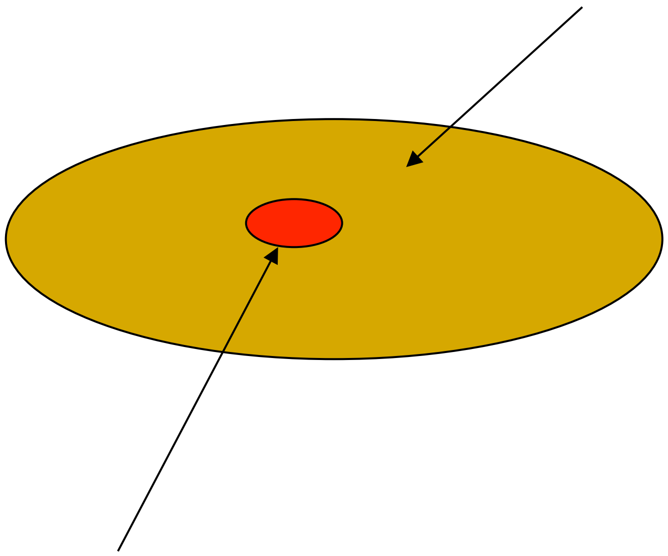
$$[\rho] = \frac{C}{m^3}$$



$$\begin{aligned} d\vec{E}(\vec{x}) &= \frac{dq}{4\pi\epsilon_0 r^2}\vec{e}_r \\ &= \frac{\rho dV'}{4\pi\epsilon_0|\vec{x}-\vec{x'}|^2}\,\vec{e}_r \\ &= \frac{\rho dV'}{4\pi\epsilon_0|\vec{x}-\vec{x'}|^3}\,(\vec{x}-\vec{x'}) \end{aligned}$$

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho dV}{r^2}\vec{u}_r = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{x'})d^3x'}{|\vec{x}-\vec{x'}|^3}(\vec{x}-\vec{x'})$$

Densita` superficiale di carica



Densita` superficiale di carica

$$\sigma = \frac{\mathrm{d}q}{\mathrm{d}A}$$

$[\sigma] = C/m^2$

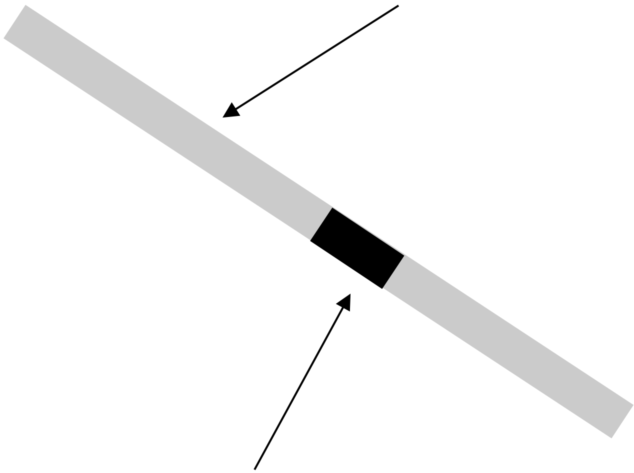
Carica totale sulla superficie

$$Q = \int_S \sigma \, \mathrm{d}A$$

$$E(\mathbf{r}) = \int_\Sigma dA' \sigma(\vec{\mathbf{r}}') \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{4\pi\varepsilon_0 \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}$$

Densita` lineare di carica

Carica totale Q



Elemento di linea dl

dq = λ dl

Densita` lineare di carica

$$\lambda = \frac{dq}{dl}$$

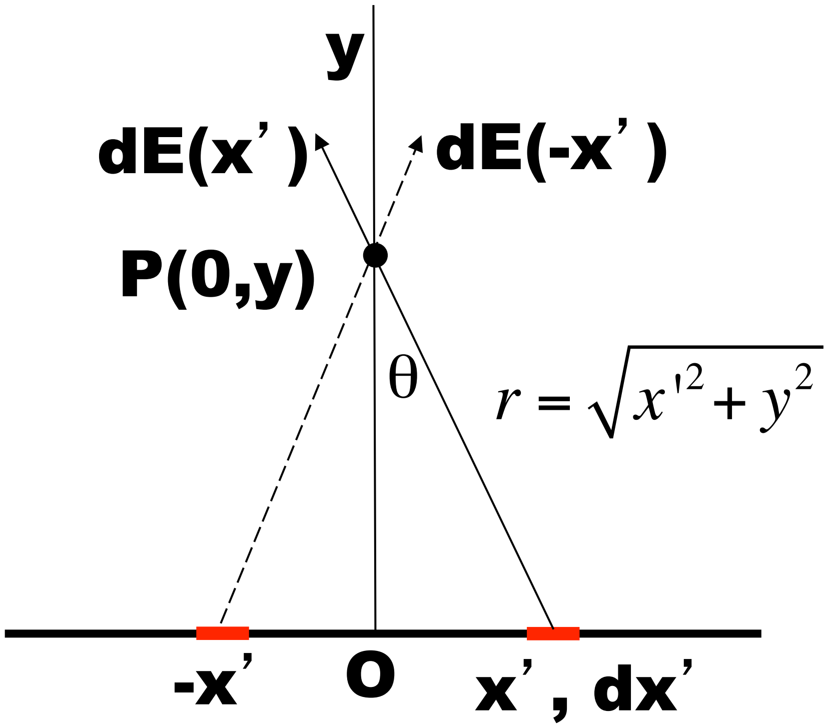
[λ]=C/m

Carica totale

$$Q = \int_l \lambda \, dl$$

$$E(\mathbf{r}) = \int_l dl' \lambda(\vec{\mathbf{r}}') \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{4\pi\varepsilon_0 \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3}$$

Esempio: campo sull'assse di un filo (lungo 2l)



$$dq = \lambda dx'$$

Densita` uniforme λ=cost.

$$d\vec{E}(0, y; x') = \frac{\lambda dx'}{4\pi\epsilon_0 r^2}\frac{\vec{r}}{r}$$

$$2\frac{\lambda dx'}{4\pi\epsilon_0 r^2}\frac{y}{r} = 2\frac{\lambda dx'}{4\pi\epsilon_0}\frac{y}{(x'^2 + y^2)^{3/2}}$$

$$y = r\cos\theta \quad x' = y\tan\theta \quad dx' = \frac{y\,d\theta}{\cos^2\theta}$$

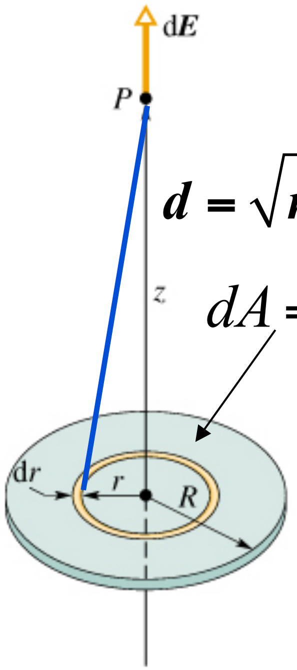
$$\int_0^{\theta_l} \cos\theta d\theta = [\sin\theta_l]_0^{\arcsin\frac{l}{\sqrt{y^2+l^2}}} = \frac{l}{\sqrt{l^2 + y^2}}$$

$$E(0,y) = \frac{2\lambda l}{4\pi\varepsilon_0}\frac{1}{y\sqrt{y^2 + l^2}} = \frac{q}{4\pi\varepsilon_0}\frac{1}{y\sqrt{y^2 + l^2}}$$

$y >> l$ E $\propto q/y^2$

$y << l$ $(l/y \rightarrow \infty)$ E $\propto \lambda/y$

Campo elettrostatico sull'asse di un disco uniformemente carico



$d = \sqrt{r^2 + z^2}$

$dA = 2\pi r dr$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 d^2}\vec{u} = \frac{\sigma dA}{4\pi\varepsilon_0 d^2}\vec{u}$$

lungo l' asse z: $(\vec{u})_z = \frac{z}{d}$

$$dE(z) = \frac{\sigma(2\pi r dr)}{4\pi\varepsilon_0 d^2}\frac{z}{d} = \frac{\sigma z}{2\varepsilon_0}\frac{rdr}{(z^2 + r^2)^{3/2}}$$

$$E(z) = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{rdr}{(z^2 + r^2)^{3/2}}$$

$$E(z) = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{rdr}{(z^2+r^2)^{3/2}} \qquad (r^2 = t, rdr = \frac{1}{2}dt)$$

$$= \frac{\sigma z}{2\varepsilon_0}\frac{1}{2}\int_0^{R^2}\frac{dt}{(z^2+t)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\frac{1}{2}\frac{1}{-\frac{3}{2}+1}\left[(t+z^2)^{-3/2+1}\right]_0^{R^2}$$

$$= -\frac{\sigma z}{2\varepsilon_0}\left[(t+z^2)^{-1/2}\right]_0^{R^2} \qquad = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{|z|} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= \pm\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{|z|}{\sqrt{z^2+R^2}}\right]$$

$$\vec{E}(z) = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{|z|}{\sqrt{z^2 + R^2}}\right] \mathrm{sign}(z)\,\vec{e}_z$$

$z/R \to +\infty$ → carica puntiforme

$$\frac{\sigma}{2\varepsilon_0}[1 - \frac{z}{\sqrt{z^2 + R^2}}] = \frac{\sigma}{2\varepsilon_0}[1 - \frac{1}{\sqrt{1 + (R/z)^2}}] \simeq \frac{\sigma}{2\varepsilon_0}[1 - (1 + \frac{R^2}{2z^2})] = \frac{\sigma R^2}{4\varepsilon_0 z^2} = \frac{Q}{4\pi\varepsilon_0 z^2}$$

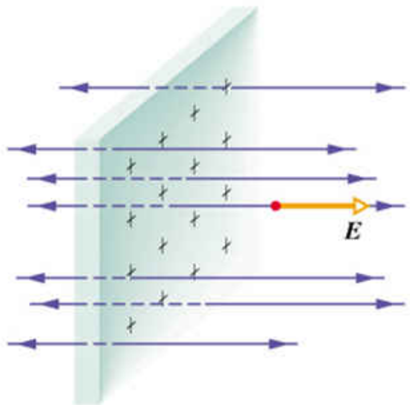
Disco visto come una carica puntiforme $Q=\pi R^2 \sigma$

Limite per $z \to 0$ **diverso per z>0 e z<0**

$$\vec{E}(z) = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{|z|}{\sqrt{z^2 + R^2}}\right] \mathrm{sign}(z)\, \vec{e}_z$$

$$\vec{E} = \pm \frac{\sigma}{2\varepsilon_0}\vec{n}$$

corrisponde a un piano infinito (z/R → 0)

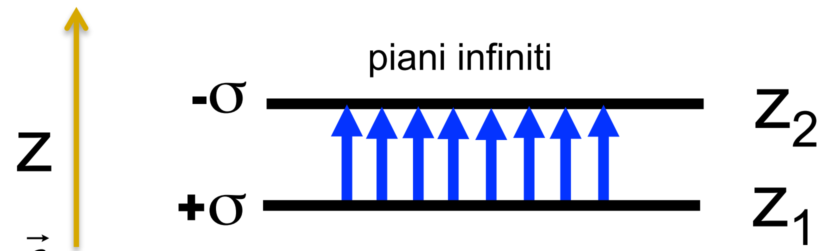


Campo **discontinuo** nell' attraversamento dello strato carico

$$\Delta\vec{E} = \vec{E}(z_+) - \vec{E}(z_-) = \frac{\sigma}{\varepsilon_0}\vec{n}$$

Esempio: Doppio strato

Usiamo il principio di sovrapposizione



$\vec{n} \equiv \vec{e}_z$

$$\vec{E}_{\text{int}} = \frac{\sigma}{\varepsilon_0}\vec{n}$$

$$\vec{E}_{\text{ext}} = \vec{0}$$

z	$\vec{E}_{+\sigma}$	$\vec{E}_{-\sigma}$	$\vec{E} = \vec{E}_{+\sigma} + \vec{E}_{-\sigma}$
$> z_2$	$\frac{+\sigma}{2\varepsilon_0}\vec{n}$	$\frac{-\sigma}{2\varepsilon_0}\vec{n}$	0
$z_1 < z < z_2$	$\frac{+\sigma}{2\varepsilon_0}\vec{n}$	$\frac{-\sigma}{2\varepsilon_0}(-\vec{n})$	$\frac{\sigma}{\varepsilon_0}\vec{n}$
$z < z_2$	$\frac{+\sigma}{2\varepsilon_0}(-\vec{n})$	$\frac{-\sigma}{2\varepsilon_0}(-\vec{n})$	0

$$\vec{E}_{\text{ext}} = \vec{0}$$

Discontinuita`

$$\pm\frac{\sigma}{\varepsilon_0}$$