

## Zedometry

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One of the LEP running dedications through years 1989 to 1995 was the precision study of the Z Boson parameters, the so called “line-shape”. The combination of the results from the four LEP collaborations, together with improved experimental performances and theoretical calculations, has lead to an impressive achievement.

### 1. Motivation

After LEP startup in 1989, the next six years of LEP running (1990-1995) were largely dedicated to precision studies of the Z Boson parameters. Collisions were delivered at several well determined center of mass energies around the Z resonance, with an always improved luminosity. Each of the four detectors (Aleph [1], Delphi [2], L3 [3], Opal [4]) achieved a set of measurements consisting in the hadronic and leptonic cross sections, the  $\tau$  polarisation asymmetries, the  $b\bar{b}$  and  $c\bar{c}$  partial widths and forward-backward asymmetries through many center of mass energies.

Because of changes in the experimental conditions, such as replacement of detector components, data had to be subdivided into further subsamples. This has resulted into something like 4 x 200 individual measurements.

Although it was realized from the very early days of LEP running that a combination of results would lead to an improved precision because of the reduced statistical error and possibly uncorrelated errors, this was only possible through a common set of agreed quantities, the “pseudo-observables” ( $M_Z$ ,  $\Gamma_Z$ ,  $\sigma_{had}^0$ ,  $R_e$ ,  $R_\mu$ ,  $R_\tau$ ,  $A_{FB}^{0,e}$ ,  $A_{FB}^{0,\mu}$ ,  $A_{FB}^{0,\tau}$ ). This presentation will concentrate on the so-called “Lineshape” variables, other aspects like the discussion of electroweak fits and results on heavy flavor asymmetries being covered by other talks in this conference [5,6].

### 2. The Z Lineshape parameters

Near the Z peak, the hadronic cross section is dominated by the Z exchange and can be expressed in terms of a Breit-Wigner shape corrected for QED effects, plus  $\gamma$  exchange and  $\gamma$ -Z interference terms :

$$\sigma_{had}(s) = \sigma_{had}^0 \frac{1}{1 + \delta_{QED}} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} + \sigma_\gamma + \sigma_{\gamma Z},$$

where the parameters are the Z mass  $M_Z$ , the width  $\Gamma_Z$  and the peak hadronic cross section  $\sigma_{had}^0$  and  $\delta_{QED} = 3\alpha(M_Z)/4\pi$ . This is illustrated in Figure 1.

The total width is related to the hadronic pole cross section by :

$$\Gamma_Z^2 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\sigma_{had}^0} = (3\Gamma_{\ell\ell} + \Gamma_{had} + N_\nu\Gamma_\nu)^2,$$

where  $\Gamma_{\ell\ell}$  is the partial width into leptons,  $\Gamma_{ee}$  ( $\Gamma_{had}$ ) the electron (hadronic) width and  $\Gamma_\nu$  the partial width into neutrinos. This introduces the parameter  $N_\nu$ , the number of neutrino families which is one of the most striking results of the even first months of LEP running in 1989 !

### 3. Statistics

Data have been collected at the Z peak energy in 1992 and 1994, while scans in energy around the Z peak have been performed in the range  $|\sqrt{s} - M_Z| < 3 \text{ GeV}$  in 1990, 1991 and

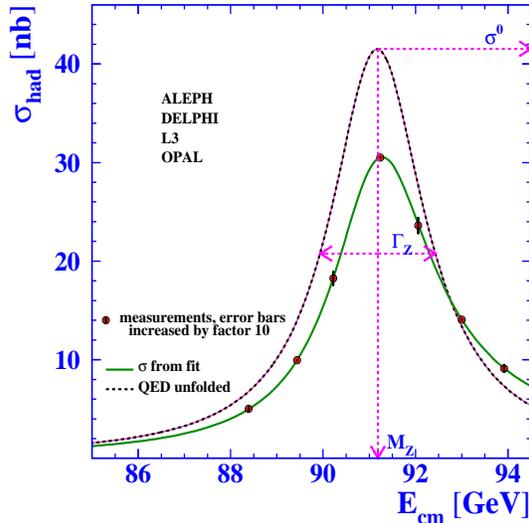


Figure 1. The hadronic cross section measurements and the Lineshape variables definition

$|\sqrt{s} - M_Z| < 1.8 \text{ GeV}$  in 1993 and 1995. More than 15 Millions of hadronic decays and 1.7 Millions of leptonic decays have been accumulated by the four LEP experiments. The breakdown of the integrated luminosity per year is shown in Table 1.

#### 4. Energy calibration

Precise knowledge of the energy of the colliding particles at the interaction point is essential for the determination of the Z resonance parameters. The uncertainty in the absolute energy scale directly affects the mass, whereas the width is only affected by the error on the difference in energy between energy points.

The determination of the average energy of the beams[7] is based on the technique of resonant depolarisation (RD) characterised by a very high precision ( $\pm 0.2 \text{ MeV}$ ) at the time of the measurement. These measurements were only available outside data taking periods, at the end of fills (typically every 10 hours). Only 40% of the off-peak luminosity was calibrated this way in the 1993 scan and 70% in the 1995 scan.

Table 1

Luminosity statistics accumulated from 1990 to 1995. In 1990 and 1991 a total of  $7 \text{ pb}^{-1}$  was recorded off peak and  $20 \text{ pb}^{-1}$  in each of the 1993 and 1995 years.

year	Beam energy range (GeV)	Integrated Lumi. ( $\text{pb}^{-1}$ )
1990 scan	[88.2, 94.2]	8.6
1991 scan	[88.5, 93.7]	18.9
1992	91.3	28.6
1993 scan	[89.4, 93.]	40.0
1994	91.2	64.5
1995 scan	[89.4, 93.]	39.8
total	[88.2, 94.2]	200.4

The extrapolation of these precise values at a particular time to the full set of data requires corrections for the time-dependence of the magnetic field in the bending dipole magnets and for the changes in energy caused by deformations of the LEP ring. These corrections were performed with a model based on direct measurements of the magnetic dipole fields (NMR probes), corrector magnet fields, beam orbit position and RF system parameters. External effects such as the variation of the leakage currents produced by TGV's trains passing by, deformations of the ring caused by terrestrial tides (Figure 2) and variations of the water level in the lake were studied throughout the years and propagated back to earlier years when understood.

#### 5. Absolute cross section normalisation

Bhabha scattering  $e^+e^- \rightarrow e^+e^-$  is used as the reference reaction to provide the absolute normalisation of other processes. The rate of such events detected in specially designed monitors is used to measure the absolute luminosity of the colliding beams, found by dividing the number of monitored events by the Bhabha cross section integrated over the acceptance:

$$L = \frac{N_{Bhabha}^{obs}}{\sigma_{Bhabha}^{acc}}$$

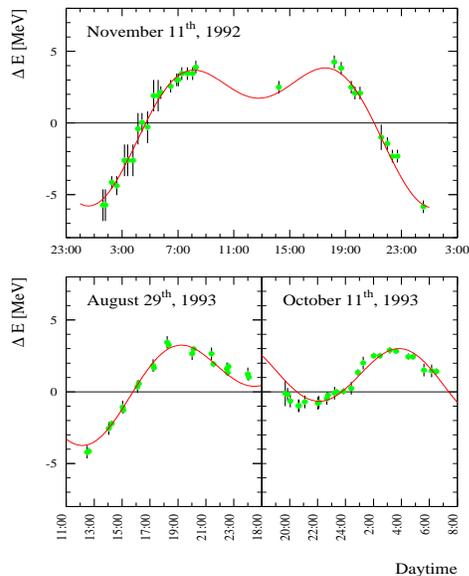


Figure 2. Comparison of measurements with the prediction of the tide model for two periods, full-moon and close to half-moon

The effect of the electroweak process  $Z \rightarrow e^+e^-$  on the reference cross section is limited by restricting the acceptance to small angles, where the Bhabha rates are dominated by  $t$ -channel photon exchange and therefore largely described by QED. The large cross section at low angles gives the further advantage of reducing the statistical uncertainty of the reference measurement. Given the high statistics accumulated each year, leading to small statistical errors on cross section measurements, the goal was to achieve a similar relative systematic precision of about one per mil.

Two generations of Luminosity detectors have been used at LEP allowing for the required precision on the Luminosity determination : before LEP, the precision of Luminosity detectors at PEP and PETRA was 2–3%. The first LEP generation has reached a precision of 0.3–0.5% with a Bhabha counting rate similar to the Z counting rate, while the second generation, starting from Autumn 1992, reached a 0.07–0.1% precision in 1995 with a counting rate up to three times the

Z counting rate.

The Bhabha cross section is expressed as :

$$\frac{d\sigma}{d\Omega} = \frac{16 (\hbar c \alpha)^2}{s} \left( \frac{1}{\theta^4} \right)$$

which, integrated over the acceptance, shows a fast decrease with the scattering angle:

$$\sigma^{\text{acc}} = \frac{1040 \text{ nb GeV}^2}{s} \left( \frac{1}{\theta_{\text{min}}^2} - \frac{1}{\theta_{\text{max}}^2} \right).$$

As a consequence, a change of the inner radial acceptance will induce a systematic shift in Luminosity for detectors at a fixed distance from the interaction point. The control of the lower radial acceptance was the challenge of the next generation detectors.

From 1992 onwards, the beam pipe diameter was reduced from 16 cm to 11 cm, allowing for a smaller minimum radius of the detectors. Aleph (in Autumn 92) and Opal (in Spring 93) installed Silicon-Tungsten (SiW) calorimeters with full azimuthal acceptance. L3 upgraded the existing detector and DELPHI STIC was operational in 1994. The characteristics of the first and second generation Luminosity detectors are given in Table 2.

Experimental errors on the Luminosity determination have thus impressively decreased with years, sometimes by as much as a factor 20, ending well below the 0.1% value ! This was made possible also by new theoretical calculations of the reference Bhabha cross section. A very fruitful collaboration with theorists allowed to reduce the theoretical uncertainty from the generator BHLUMI to only 0.061%. As all analyses are based on the same calculations, this error is common to all results and has to be carefully taken into account. The evolution with years of the Luminosity experimental relative precision is given in Table 3.

## 6. Determination of errors and results

Each year, when new results became available from the individual collaborations, the LEP Electroweak Working group performed a careful combination of results [8], with particular attention to the error matrix which had to take into account

Table 2  
Key features of the two generation Luminosity detectors

First generation	distance from IP(m)	$R_{min}$ (cm)	$R_{max}$ (cm)	$\theta_{min}$ (mrad)	$\theta_{max}$ (mrad)	Technology
ALEPH LCAL	2.7	10.	52.	45.	190.	Lead + prop.wire chambers
DELPHI SAT	2.5	10.	40.	43.	135.	Lead + Scint.fibers
L3 BGO	2.8	6.8	19.	25.	70.	BGO crystals
OPAL FD	2.4	11.5	29.	48.	120.	Lead + scintil.

Second generation	$R_{min}$ (cm)	$R_{max}$ (cm)	$\theta_{min}$ (mrad)	$\theta_{max}$ (mrad)	Cross section (nb)	Technology
ALEPH SiCAL	6.1	14.5	24.	48.	84.	Si W calorimeter
DELPHI STIC	7.0	18.0	31.	185.	65.	Lead + Scint. tiles
L3 SLUM	7.6	15.4	29.	58.	50.	BGO + silicon plans
OPAL SiW	6.1	14.1	25.	59.	90.	Si W calorimeter

Table 3  
Evolution of experimental relative precision of Luminosity detectors

	1990	1991	1992	1993	1994	1995
ALEPH LCAL,SiCAL	0.47%	0.37%	0.15%	0.067%	0.073%	0.080%
DELPHI SA,STIC	0.80%	0.50%	0.38%	0.28%	0.09%	0.09%
L3 BGO,SLUM	0.50%	0.50%	0.50%	0.086%	0.064%	0.068%
OPAL FD,SiW	0.70%	0.45%	0.41%	0.033%	0.033%	0.034%

uncorrelated and correlated errors, between experiments, between years and between energies.

The common set of nine parameters was extracted by each experiment using fits based on ZFITTER and TOPAZ0 programs[9]. Under the assumption of lepton universality, the nine parameters set reduces to a five parameters one ( $M_Z, \Gamma_Z, \sigma_{had}^o, R_\ell, A_{FB}^{0,\ell}$ ). The chosen parameters are almost uncorrelated and thus well adapted to fitting and averaging procedures. Table 4 gives the extracted values for the five parameters fit and the correlation matrix. The only sizeable correlations are between ( $\Gamma_Z$  and  $\sigma_{had}^o$ ) and ( $R_\ell$  and  $\sigma_{had}^o$ ). They contain the sensitivity to the Standard Model parameters  $M_H$  and  $\alpha_s$ .

### Common uncertainties

The nine (five) parameters from each experiment are then combined using a very sophisticated description of the common uncertainties. Those uncertainties arise from several sources

with the most important ones being the calibration of the LEP beam energy, the theoretical error on the calculation of the small-angle Bhabha cross section (as explained in section 5) and theoretical uncertainties on the large-angle Bhabha cross section.

The beam energy uncertainty contributes an uncertainty of  $\pm 1.7$  MeV to  $M_Z$  and  $\pm 1.2$  MeV to  $\Gamma_Z$ . In addition, the uncertainty in the centre-of-mass energy spread of about  $\pm 1$  MeV contributes  $\pm 0.2$  MeV to  $\Gamma_Z$ . The contribution to other parameters is negligible.

All four collaborations use BHLUMI 4.04[10], the best available Monte Carlo generator of small-angle Bhabha scattering to compute the precise acceptance of their luminosity detectors. Therefore significant correlations on the scale of the measured cross sections are due to the uncertainty on this common theoretical calculation. The relative error of 0.061% on the theoretical Bhabha cross section would directly translate into

Table 4  
 Extracted values and correlation matrix for a five parameter fit including lepton universality.

		$M_Z$	$\Gamma_Z$	$\sigma_{\text{had}}^o$	$R_\ell$	$A_{\text{FB}}^{0,\ell}$
$M_Z$ [GeV]	$91.1876 \pm 0.0021$	1.				
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	-0.023	1.			
$\sigma_{\text{had}}^o$ [nb]	$41.540 \pm 0.037$	-0.045	-0.297	1.		
$R_\ell$	$20.767 \pm 0.025$	0.033	0.004	0.183	1.	
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$	0.055	0.003	0.006	-0.056	1.

an uncertainty on the hadronic peak cross section of 25 pb. A recent calculation of the contribution of light pairs[11] has been explicitly corrected for by Opal, reducing its theoretical luminosity error to 0.054%. This value is taken as correlated with the other three experiments, sharing among themselves a mutual correlated error of 0.061%.

The contribution of  $t$ -channel diagrams and the  $s - t$  interference in  $Z \rightarrow e^+e^-$  leads an additional theoretical uncertainty estimated to be  $\pm 0.024$  on  $R_e$  and  $\pm 0.0014$  on  $A_{\text{FB}}^{0,e}$ .

Other common theory uncertainties have been examined, arising from QED radiative corrections, residual Standard Model dependencies and ambiguities in the exact definition of the fitted observables. Putting all sources together, the overall theoretical errors are  $\pm 0.3$  MeV on  $M_Z$ ,  $\pm 0.2$  MeV on  $\Gamma_Z$ ,  $\pm 0.008$  nb on  $\sigma_{\text{had}}^o$ ,  $\pm 0.004$  on each  $R_\ell$  and  $\pm 0.0001$  on each  $A_{\text{FB}}^{0,\ell}$ .

## Final results

The final combined results of the five parameters fit are shown in figures 3 to 6, including  $R_\ell$  and  $A_{\text{FB}}^{0,\ell}$  which were not specifically discussed here.

## 7. Derived quantities

To check whether the invisible width (decay width into invisible particles) is completely explained by decays into the three neutrinos, the “number of neutrino species”  $N_\nu$  is calculated according to :

$$R_{\text{inv}} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = N_\nu \left( \frac{\Gamma_\nu}{\Gamma_{\ell\ell}} \right)_{SM}$$

Assuming lepton universality, the measured value of  $R_{\text{inv}}$  is  $5.942 \pm 0.0016$ . The Standard Model value for the ratio of the partial widths to neutrinos and charged leptons is  $1.9912 \pm 0.0012$ , giving the corresponding number of neutrino species to be :

$$N_\nu = 2.9841 \pm 0.0083.$$

The evolution of the error on this quantity is shown in Figure 7 illustrating the impressing improvement throughout the years.

## 8. Conclusion

Using more than 17 Millions Z decays analysed by the four LEP collaborations around the Z resonance over six years, corresponding to an integrated luminosity of  $200 \text{ pb}^{-1}$ , the LEP community has achieved an impressively precise determination of the Z lineshape parameters. The derived quantity  $N_\nu$  has been extracted, ruling out expectations based on other than three neutrino generations as illustrated in Figure 8.

## Acknowledgements

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## REFERENCES

1. Aleph collaboration, *Z. Phys. C* **48**, 365 (1990), *Z. Phys. C* **53**, 1 (1992), *Z. Phys. C* **60**, 71 (1993), *Z. Phys. C* **62**, 539 (1994), *Eur. Phys. J. C* **14**, 1 (2000).
2. Delphi collaboration, *Nucl. Phys. B* **367**, 511 (1991), *Nucl. Phys. B* **417**, 3 (1994), *Nucl.*

- Phys. B* **418**, 403 (1994), *Eur. Phys. J. C* **16**, 371 (2000).
3. L3 collaboration, *Z. Phys. C* **51**, 179 (1991), *Phys. Rept.* **236**, 1 (1993), *Z. Phys. C* **62**, 551 (1994), *Eur. Phys. J. C* **16**, 1 (2000).
  4. Opal collaboration, *Z. Phys. C* **52**, 175 (1991), *Z. Phys. C* **58**, 219 (1993), *Z. Phys. C* **61**, 19 (1994), *Eur. Phys. J. C* **19**, 587 (2001),
  5. E.Lançon, *these proceedings* .
  6. W.Liebig, *these proceedings* .
  7. L.Arnaudon *et al.*, *Phys. Lett. B* **307**, 187 (1993), CERN-SL/93-21, R.Assman *et al.*, *Z. Phys. C* **66**, 567 (1995), *Eur. Phys. J. C* **6**, 187 (1999)
  8. The LEP collaborations and the LEP Electroweak working group, CERN/PPE/93-157, CERN/PPE/94-187, CERN/PPE/95-172, CERN-PPE/96-183, CERN-PPE/97-154, CERN-EP/99-15 , CERN-EP-2000-016 , CERN-EP-2001-021, CERN-EP-2001-098
  9. ZFITTER: D.Bardin *et al.*, *Z. Phys. C* **44**, 493 (1989), *Nucl. Phys. B* **351**, 1 (1991), *Phys. Lett. B* **255**, 290 (1991), *Comput. Phys. Commun.* **133**, 229 (2001); TOPAZ0: G.Montagna *et al.*, *Nucl. Phys. B* **401**, 3 (1993), *Comput. Phys. Commun.* **76**, 328 (1993), *Comput. Phys. Commun.* **93**, 120 (1996), *Comput. Phys. Commun.* **117**, 278 (1999);
  10. S.Jadach *et al.*, *Comput. Phys. Commun.* **102**, 229 (1997);
  11. G.Montagna *et al.*, *Nucl. Phys. B* **547**, 39 (1999), *Phys. Lett. B* **459**, 649 (1999);

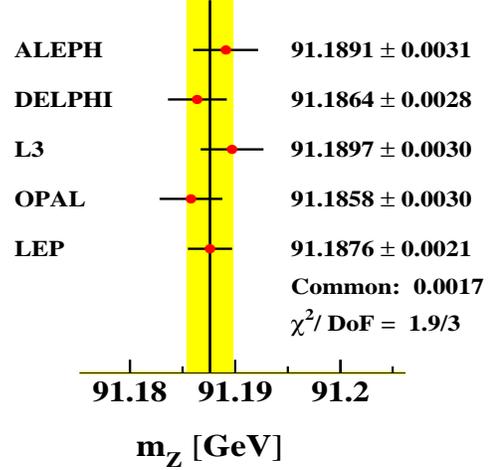


Figure 3. Combined measurement of the Z mass. The common error comes from the beam energy calibration.

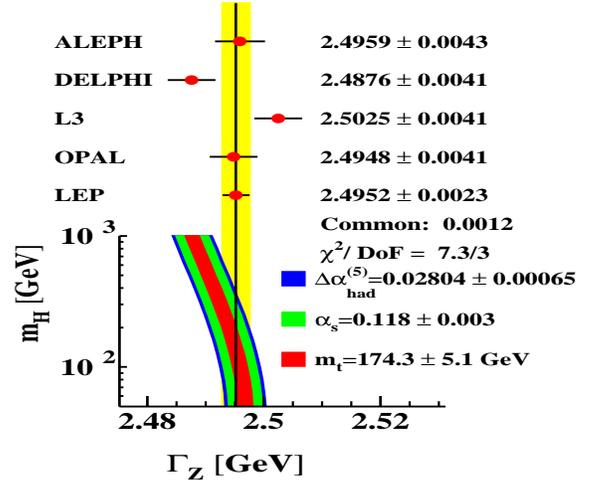


Figure 4. Combined measurement of the Z width. The common error comes from the beam energy calibration.

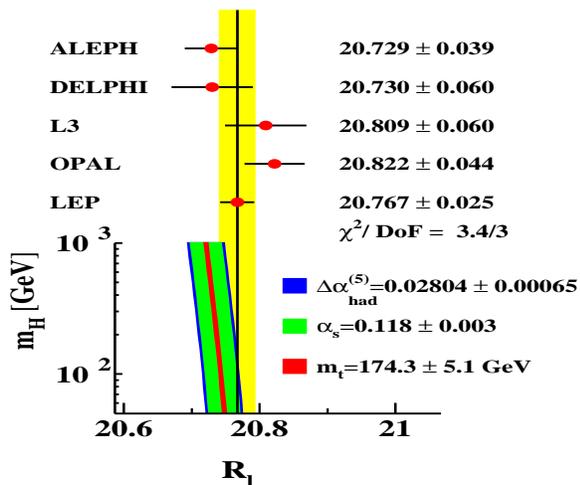


Figure 5. Combined measurement of  $R_\ell$ , the hadronic to leptonic width ratio.

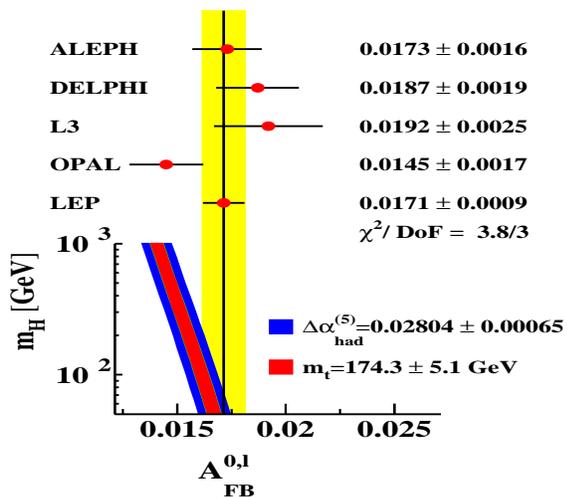


Figure 6. Combined measurement of  $A_{\text{FB}}^{0, \ell}$ , the Forward-Backward leptonic asymmetry.

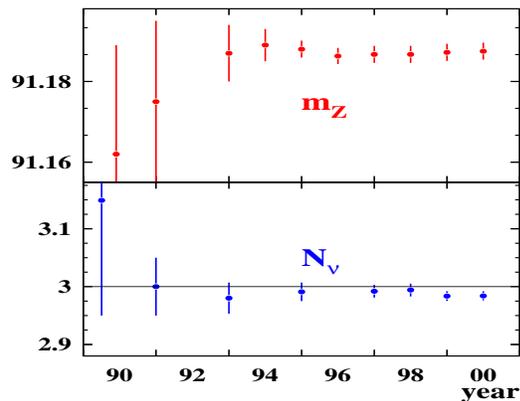


Figure 7. Variation with the data taking year of  $M_Z$  and  $N_\nu$ . The improved statistical power and more sophisticated analysis are reflected in the decreasing size of error bars.

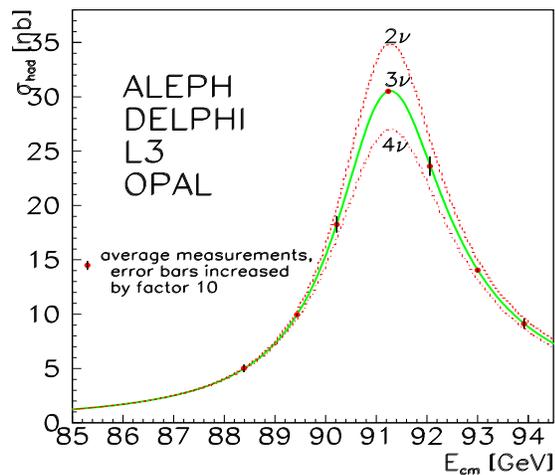


Figure 8. The hadronic cross section measured by the four experiments, together with predictions assuming 2,3 and 4 neutrino families. This result clearly favours the three neutrino hypothesis!